

PEAK HEAT FLUX AND TEMPERATURE DIFFERENCE IN NUCLEATE BOILING OF LIQUEFIED GASES

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INTRODUCTION

Two-phase phenomena are encountered frequently in cryogenic systems. Since the thermal diffusivity and kinematic viscosity of the liquids under consideration (and their vapors) are low, the critical value of the Rayleigh or Reynolds number is easily exceeded. Consequently, turbulent conditions prevail and a complicated flow pattern results. A special type of two-phase flow is involved in nucleate boiling which is characterized by the appearance of the vapor phase in the form of single bubbles. It may be caused by heat currents leaking through the outer boundaries of the system (if a conventional type of insulation is used), or by a heat input required for the operation of the system. A thermal boundary layer of superheated liquid is built up near the confining or heat emitting solid surfaces from which the bubbles grow. Since the thermal conductivity of the vapor is much lower than that of the liquid, heat flows to the growing bubbles via the conducting liquid in between the bubbles. Further details on the mechanisms involved have been given in the literature[¹].

The surface excess temperature $\Delta T = T_w - T_s$ accompanying nucleate boiling often is compatible with cryogenic requirements and allows satisfactory operation. However, if the heat flux delivered from the wall is so large that the resulting vapor volume no longer permits a liquid path of sufficiently large thermal conductance, the mode of nucleate boiling breaks down and film boiling will be established, accompanied by a large surface excess temperature. This latter boiling mode sometimes may be useful, for instance, if superconductivity is to be destroyed thermally by exceeding the critical temperature[²]. However, film boiling is usually avoided. Therefore it is important for the design and operation of cryogenic systems to know the conditions determining the two fundamentally different boiling regimes, in particular the peak nucleate boiling heat flux q_D ("maximum flux," "burnout flux") and the accompanying temperature difference $(\Delta T)_D$. The two quantities are dealt with subsequently. We restrict ourselves to a one-component system and normal cryogenic liquids.

PEAK HEAT FLUX

Under steady state conditions of the system, the total rate of growth of the vapor phase is the product of the number of active centers i at the surface and the rate of growth of the vapor domain at a single center. A total volume flow rate of vapor

$$\dot{V}_{\text{tot}} = i \cdot V_v / \tau \quad (1)$$

results if the vapor volume V_v is produced at a single center with a frequency $1/\tau$. With rising heat flux the number of centers increases until a maximum value will be reached at the peak flux. It may be convenient to introduce a coverage factor ζ defined as the ratio of the surface covered by bubbles to the total surface area A . Then the number of centers may be expressed in terms of ζ by noting that the product of ζ with the total surface area is equal to the number of centers multiplied by a mean bubble cross section, averaged in time and over the surface. At the peak heat flux, the coverage factor approaches a maximum value ζ_p , ($\zeta_p < 1$). As soon as ζ_p is known, the number of centers can be eliminated from (1). The other unknown quantities of this equation are closely related to bubble kinematics and dynamics.

As an example, we may consider free-convection nucleate boiling on a horizontal plate at the bottom of a vessel filled with saturated liquid. Under steady state conditions, if there were a continuous emission of bubbles, the average mass flow rate of vapor from a single center would have to be accounted for by an equal mass flow rate of liquid to the center. At the center, however, the volume of a departing bubble must be replaced by liquid. Due to the excess mass flow of liquid to the center, a coverage factor of the order 10^{-1} is expected at the peak heat flux. By changing the meaning of ζ_p and defining it by means of the maximum bubble cross section A^* , we arrive at the relation

$$\zeta_p = i_p(A^*/A) \quad (2)$$

(For first-order estimates $\zeta_p \approx \frac{1}{4}$ may be inserted.) The value of the peak coverage factor is expected to be approximately constant in different cases since similar considerations apply to various nucleate boiling systems.

At high population densities, and thus also at q_p , the total amount of heat delivered from the surface is transported indirectly to the bulk liquid via the boiling process. The volume flow rate of vapor due to the peak flux q_p for this vapor-liquid exchange mechanism^[1] will be

$$(\zeta_p/\tau)(A/A^*)\dot{V}_v = q_p A / L\rho_v \quad (3)$$

Exact values of the quantities on the left-hand side of (3) have to be determined experimentally due to the difficulties of predicting turbulent flow phenomena. A qualitative discussion, however, might be useful. The unknown ratio \dot{V}_v/A^* is proportional to the maximum bubble radius R^* . The latter has a value of the order of H , the thickness of the thermal boundary layer^[3]. Therefore q_p will be proportional to H/τ . If H is small, i.e., the (mean) temperature gradient large, the bubbles will grow rapidly. Since experimental work on cryogenic boiling has recently been summarized by Richards *et al.*^[4], only a few comments on the peak heat flux will be given here.

Subcooled Boiling

If a subcooled liquid is exposed to a high heat flux, an estimate of q_p may be obtained by means of an approximate relation given by Forster *et al.*^[1] The quantity $R^{*3}(T_w - T_l)/\tau$ has been found to be approximately constant. Accordingly

$$q_p \sim [R^{*3}(T_w - T_l)]^{-1} \quad (4)$$

or $q_p \sim H^{-2}$. Of course this estimate cannot be expected to hold true for the whole range of subcooling and for all geometrical arrangements and operating conditions.

Natural Convection Pool Boiling of a Saturated Liquid

This type of nucleate boiling occurs frequently in cryogenic containers filled with normal liquids. The bubbles do not collapse as in subcooled boiling, but are carried into

the bulk liquid. The equation given by Kutateladse^[5] satisfactorily describes the peak flux on a horizontal plate or a similar configuration

$$q_p = K_k L \rho_v^{1/2} [g \sigma (\rho_l - \rho_v)]^{1/4} \quad (5)$$

where from experiments $K_k = 0.16$.

Experimental results indicate^[8] that small and multi- g accelerations may be accounted for satisfactorily by inserting the proper multiple of g into (5).

Forced Convection (Saturated Liquid)

At high velocities, the thickness of the thermal boundary layer becomes smaller than in free-convection systems. Therefore, the peak flux is expected to increase, provided the quality is low. At high qualities, however, the chances are great that bubbles approaching the solid surface reduce the thermal conductance of the liquid layer adjacent to the wall. Consequently, the peak flux is lowered and transition to film boiling may occur easily. In the particular case of swirling flow, (5) might be used by replacing g with the appropriate acceleration term ($\omega^2 r$).

Complex Natural Convection Systems

The decrease of the peak flux is most remarkable within confined channels connected to a large reservoir of liquid while motion is maintained due only to natural convection, (e.g., cryogenic cooling of electromagnets). In this case, q_p has been found^[7] to be considerably smaller than the value calculated from (5).

In helium dewars with conventional insulation, the bubbles rising from the container walls induce motion of the liquid ("simulated forced convection")^[8]. The peak flux of a heater mounted within the bath will then be higher than the values computed from (5); however, large scatter is observed which might be due to the presence of the bubbles in the liquid.

Electrical Fields

As in the case of "simulated forced convection" the peak flux may be increased by electrical fields^[9]. In the experiments reported so far, q_p was not increased by an order of magnitude. Therefore, conservative calculations may be based upon (5); however, the factors which may lower the peak flux should be considered carefully.

PEAK TEMPERATURE DIFFERENCE

Whereas the peak flux in different systems has been found to depend on the state of the bulk liquid, the peak temperature difference $(\Delta T)_p$ is much more closely related to surface properties and thermodynamics of phase change, and, to a first order approximation, independent of the conditions of the bulk liquid. The temperature difference for a given surface (given roughness and contamination) is fairly constant at high bubble population densities. Remarkable differences in the $(\Delta T)_p$ values are encountered when the microroughness of the surface is changed artificially^[10]. It is expected, however, that the surface properties of cryogenic nonbrittle materials will not vary so much as those of artificially created surfaces or of other construction materials which may be changed drastically by liquids, such as water from deposits, corrosion, or other reactions. For practical cryogenic purposes, therefore, a boiling correlation might be satisfactory if the influences of the physical properties and pertinent operation parameters are accounted for properly. To obtain the peak value $(\Delta T)_p$ two approaches are taken. One way consists of inserting the appropriate peak heat flux into a given general boiling

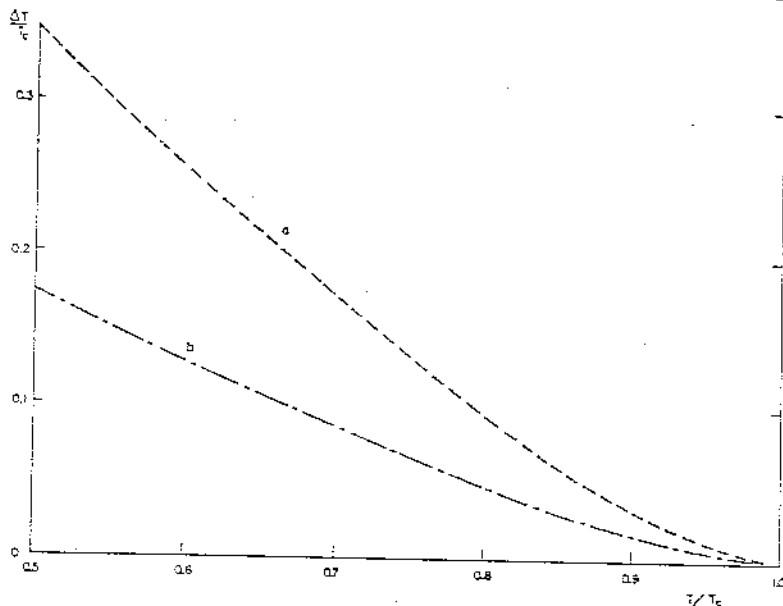


Fig. 1. Reduced temperature differences. Curve (a)— $(\Delta T)_{\max}/T_c$ for constant pressure according to (9); curve (b)—estimated boiling peak temperature difference $(\Delta T)_p/T_c$.

correlation and solving for $(\Delta T)_p$. The second approach taken will be based solely on thermodynamical considerations.

In determining the peak temperature difference by the first method, the choice of the peak flux equation is not very critical for a given surface since ΔT does not vary considerably in changing the flux. Therefore the peak value of (5) will be taken as the reference flux. As a general boiling correlation, the equation proposed by Forster and Greif^[1], will be chosen since it was successful in predicting the heat transfer in boiling of widely different liquids in the low-pressure range.

$$\bar{q} = 1.2 \cdot 10^{-3} \frac{\alpha_l C_l \rho_l T_s}{L \rho_v \sigma^{1/2}} \left[\frac{C_l T_s \alpha_l^{1/2}}{(L \rho_v)^2} \right]^{1/4} \left(\frac{\rho_l}{\mu_l} \right)^{5/8} \left(\frac{\mu_l C_l}{K_l} \right)^{1/3} \Delta p^2 \quad (6)$$

According to this equation, ΔT is predicted to be fairly constant at high flux values; however, an approximation which is obtained by making use of the Clausius-Clapeyron equation can be handled more easily. Though ΔT , for a given \bar{q} , quantitatively differs somewhat from the value evaluated by means of (6), this approach seems to be sufficient for our purposes. Replacing Δp by $L \rho_v \Delta T / T_s$ where $(1/\rho_l \ll 1/\rho_v)$, we obtain a flux

$$\bar{q} \approx 1.2 \cdot 10^{-3} \frac{\alpha_l C_l \rho_l}{T_s} \left[\frac{L \rho_v}{\sigma} \right]^{1/2} (C_l T_s \alpha_l^{1/2})^{1/4} \left(\frac{\rho_l}{\mu_l} \right)^{5/8} \left(\frac{\mu_l C_l}{K_l} \right)^{1/3} \Delta T^2 \quad (7)$$

Combining (5) and (7) we obtain

$$(\Delta T)_p \approx \left(\frac{400}{3} \right)^{1/2} g^{1/8} \frac{L^{1/4} (\sigma T_s)^{3/8} (\rho_l - \rho_v)^{1/8}}{\rho_l^{1/2} \alpha_l^{1/4} C_l^{5/8}} \left(\frac{\mu_l C_l}{K_l} \right)^{7/48} \quad (8)$$

Equation (8) provides a first-order approximation at low pressures or at low values of the reduced temperature $T_r = T_s/T_c$, ($T_c =$ critical temperature).* At high pressures or saturation temperatures the $(\Delta T)_p$ values predicted are too high. (We note that this method

* Figures which show this approximation for various fluids are available, but have not been included in this presentation.

provides a critical check of general boiling correlations which have been proposed so far or which will be proposed in the future.) The physical properties are those of the superheated liquid or the saturated vapor^[1]. They may be expressed in terms of dimensionless parameters of state: $T_r = T/T_c$, $P_r = p/p_c$, or $v_r = v/v_c$. Since substitutions of this kind lead to rather lengthy expressions, a shorter approximation is much more desirable.

Our second approach is based upon a crude estimate of the possible metastable states which may be reached in the two-phase region. For a given pressure (or a given value of p_r) there exists a maximum reduced temperature difference $(\Delta T)_{\max}/T_c$, at the minimum of the metastable isotherms in a p - v diagram, which might be attained with a very clean system. One of the simplest equations of state is that due to van der Waals

$$p_r = [8T_r/(3v_r - 1)] - (3/v_r^2) \tag{9}$$

Equation (9) may be useful for the purposes of a first-order estimate though quantitative agreement cannot be expected. At the minimum of the isotherms we have from (9), upon differentiation and minimization with respect to v_r , $T_r^* = (3v_r - 1)^2/4v_r^3$. Since the vapor pressure can be evaluated, the values of $(\Delta T)_{\max}/T_c = [T_r^* - (T_r)_{\text{sat}}]$ are readily available. The latter have been plotted in Fig. 1 as curve (a).

So far a constant pressure system has been assumed; however, an excess pressure Δp is required for bubble growth. Assuming tentatively that the peak temperature difference, at $p + \Delta p$, is equal to the equilibrium temperature difference associated with Δp , along the vapor-pressure curve, we arrive at the reduced values $(\Delta T)_{p}/T_c$ plotted in Fig. 1 as curve (b). It is convenient to approximate this curve by a relation of the form

$$(\Delta T)_{p}/T_c = a(1 - T/T_c)^m \quad \text{when} \quad \sim 0.56 < T/T_c < 1 \tag{10}$$

where a and m are constants. Neglecting small deviations, we may insert $a \approx \frac{1}{2}$ and $m \approx \frac{2}{3}$.

Of course it cannot be expected that this procedure give quantitative agreement with experimental values since neither van der Waals' equation nor the assumption introduced account correctly for the behavior of the real system. One can expect, however, for a given surface, that the $(\Delta T)_{p}/T_c$ values of different substances should show better agreement the closer the theorem of corresponding states is satisfied. Fortunately, the theorem is obeyed surprisingly well by Ne, A, Kr, Xe, N₂, and O₂^[11].

NOTATION

- a = constant, defined by (10)
- A = surface area
- A^* = maximum bubble cross section
- C = specific heat at constant pressure
- g = acceleration due to gravity
- H = thickness of thermal boundary layer
- i = number of active nucleating sites
- K = thermal conductivity
- K_k = constant defined by (5)
- L = latent heat of vaporization
- m = constant, defined by (10)
- p = pressure
- q = heat flux (heat flow rate per unit surface area)
- r = radius
- R^* = maximum bubble radius
- T = temperature (T_w , T_s wall and saturation temperature, respectively)
- $\Delta T = T_w - T_s$
- u = bubble rise velocity
- v = specific volume

- V_v = bubble volume
 \dot{V}_{tot} = total volume flow rate of vapor
 α = thermal diffusivity
 ζ = coverage factor
 μ = dynamic viscosity
 ω = angular velocity of fluid
 ρ = density
 σ = surface tension
 τ = bubble period

Subscripts

- c = critical point
 l = liquid
 p = peak quantities
 r = reduced parameters (e.g., $T_r = T/T_c$, asterisk: minimum of metastable isotherms)
 v = vapor

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