Reprinted from ADVANCES IN CRYOGENIC ENGINEERING,
Volume 8 (1963)
Plenum Press, 227 W. 17 Street, New York 11, N.Y.

J–3

NATURAL CONVECTION FILM BOILING ON A SPHERE

T. H. K. Frederking

University of California
Los Angeles, California

and

J. A. Clark

University of Michigan
Ann Arbor, Michigan

INTRODUCTION

Immersing a solid into a liquid produces film boiling if the surface temperature is above the Leidenfrost point. Aside from cryogenic applications, metallurgists have tested the cooling abilities of liquids by dropping a hot sphere or other bodies into a liquid bath[1]. The boundary layer type of film boiling can be easily analyzed for steady state conditions if the film is laminar. This analysis also may be applied to a transient test if the change in enthalpy of the vapor film is much smaller than that of the solid, as it is in the tests described later. Therefore when "slow" cooling takes place with respect to the vapor film, quasi-stationary film boiling may be assumed to occur and be so described analytically. An analysis is presented in which a diffusion approximation is made by linearizing the governing equations. The approximate result serves as a guide to a generalized correlation of film boiling heat transfer data.

ANALYSIS

We consider a sphere in an infinite medium of pure saturated liquid. The surface excess temperature of the sphere is assumed to be sufficiently large to create a vapor film which moves upward. We further assume constant physical properties, negligible dissipation, incompressible fluid, smooth interface, isothermal wall, and we neglect radiation. Provided the film thickness is small compared to the radius of the sphere, the boundary layer type of analysis may be applied to our simplified model (coordinate system of Fig. 1), writing the basic equations for the vapor as

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\frac{\rho_l - \rho}{\rho}\sin\frac{x}{R} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y} \tag{3}$$

81

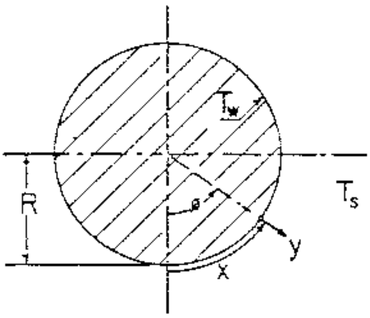

Fig. 1. Geometrical configuration and coordinates.

To complete the description of the total boundary layer, a further equation of motion has to be written for the liquid adjacent to the film replacing the vapor properties by liquid properties (subscript l, e.g., $\rho \to \rho_l$).

Boundary Conditions:

$$\begin{aligned} &\text{at } y = 0 \qquad u = v = 0 \\ &\text{as } y \to \infty \qquad u \to 0 \end{aligned} \tag{4}$$

At the vapor–liquid interface the following conditions have to be fulfilled:

(1) *Interfacial Velocity:*

$$u_i = u_{i,l} \tag{5}$$

(2) *Interfacial Shear Stress*

$$\tau_i = \tau_{i,l} \qquad \text{where} \qquad \tau_i = \mu \frac{(\partial u/\partial y)_i}{g_0} \tag{6}$$

(3) *Interfacial Mass Flow*

$$dw_i = dw_{i,l} \qquad \text{where} \qquad dw_i = \rho \frac{d}{dx}\left(\int^{A_c} u\, dA_c\right)dx \tag{7}$$

(4) *Interfacial Heat Flow Rate*

$$k\, dA[-(\partial T/\partial y)_i] = h_{fg}\, dw_i \tag{8}$$

The analysis is greatly simplified if inertia and energy convection are negligible and their effects dropped from the equations, thus linearizing them. These effects are accounted for reasonably well if the latent heat of vaporization in the final result is replaced by an "effective" heat $h'_{fg} = h_{fg} + 0.5c_p\Delta T$. If the system pressure is sufficiently far below the critical point we have $\mu\rho/\mu_l\rho_l \ll 1$. This ratio characterizing the hydrodynamical conditions at the interface has a secondary influence on the heat transfer. Therefore it is convenient to make use of the simple limiting solution for $\mu\rho/\mu_l\rho_l = 0$, noting that corrections accounting for finite values of the ratio may be taken from results valid for the flat plate. Inserting $\mu\rho/\mu_l\rho_l = 0$, we obtain a parabolic velocity profile from (2), as

$$\frac{u}{\bar{u}} = 6\left[\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right] \tag{9}$$

where

$$\bar{u} = \frac{\delta^2}{12}\,\frac{g(\rho_l - \rho)}{\nu\rho}\sin\frac{x}{R} \tag{9a}$$

Equation (3) yields a linear temperature profile

$$\frac{T - T_s}{T_w - T_s} = 1 - \frac{y}{\delta} \tag{10}$$

In our special case the hydrodynamical interface conditions have become simple boundary conditions. Equation (8) for the interfacial heat current allows evaluation of the film thickness making use of (7), (9), and (10), and noting that $dA = 2\pi R \sin(x/R)\, dx = 2\pi R^2 \sin\phi\, d\phi$, and $dw_f = \rho(d/dx)[\bar{u}\delta 2\pi R \sin(x/R)]\, dx$.
Then (8) may be rewritten as

$$\frac{96}{\mathrm{Ra}} \frac{c_p \Delta T}{h_{fg}} = \frac{\delta/R}{\sin\phi} \frac{d}{d\phi}\left[\left(\frac{\delta}{R}\right)^3 \sin^2\phi\right] \tag{11}$$

By means of the substitution

$$z = \left(\frac{\delta}{R}\right)^4 \frac{\mathrm{Ra}}{96} \frac{c_p \Delta T}{h_{fg}} \tag{12}$$

(11) becomes

$$\frac{dz}{d\phi} + \frac{8}{3} z \cot\phi - \frac{4}{3} \frac{1}{\sin\phi} = 0 \tag{13a}$$

The solution of this ordinary differential equation is

$$z = \frac{4}{3 \sin^{8/3}\phi}\left(\int^{\phi} \sin^{5/3}\phi\, d\phi + C\right) \tag{13b}$$

The constant C vanishes because δ has to be finite at $\phi = 0$. Thus

$$\frac{\delta}{R} = 2\left(\frac{8 c_p \Delta T}{\mathrm{Ra} h_{fg}}\right)^{1/4} \left(\frac{\int_0^{\phi} \sin^{5/3}\phi\, d\phi}{\sin^{8/3}\phi}\right)^{1/4} \tag{14}$$

The differential heat flow rate at the wall is

$$dq = (q/A)_0\, dA = k[-(\partial T/\partial y)_0] 2\pi R^2 \sin\phi\, d\phi \tag{15a}$$

and if we define the function $q_0(\phi)$ as

$$q_0(\phi) \equiv \frac{dq}{d\phi} = -k\left(\frac{\partial T}{\partial y}\right)_0 2\pi R^2 \sin\phi = k \Delta T \pi \frac{D}{2}\left(\frac{\mathrm{Ra} h_{fg}}{8 c_p \Delta T}\right)^{1/4} f_s(\phi) \tag{15b}$$

where

$$f_s(\phi) = \sin^{5/3}\phi \left(\int_0^{\phi} \sin^{5/3}\phi\, d\phi\right)^{-1/4} \tag{15c}$$

then the heat removed from the entire sphere is found by integration as

$$q_T = D k \Delta T \pi \frac{(2)^{1/4}}{4}\left(\mathrm{Ra} \frac{h_{fg}}{c_p \Delta T}\right)^{1/4} \int_0^{\pi} f_s(\phi)\, d\phi \tag{16}$$

In dimensionless notation, the Nusselt number may be written as

$$\mathrm{Nu} = \frac{q_T}{\pi D^2 \Delta T} \frac{D}{k} = \frac{(2)^{1/4}}{4} \pi \bar{f}_s \left(\mathrm{Ra} \frac{h_{fg}}{c_p \Delta T}\right)^{1/4} \tag{17a}$$

83

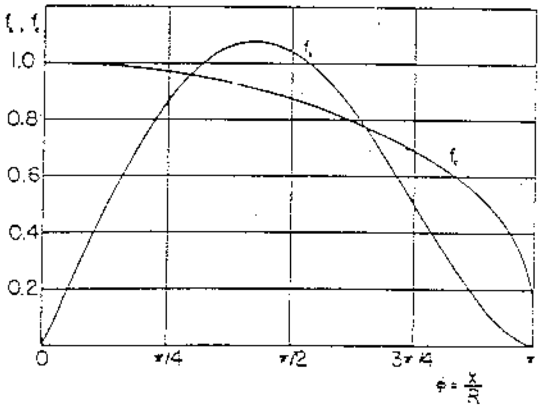

Fig. 2. Dimensionless functions f_s (sphere) and f_c (cylinder) representing local film boiling heat transfer.

Since

$$\bar{f}_s = \frac{1}{\pi}\int_0^\pi f_s(\phi)\,d\phi = 0.627$$

we obtain

$$\mathrm{Nu} = 0.586\left(\mathrm{Ra}\,\frac{h_{fg}}{c_p \Delta T}\right)^{1/4} \tag{17b}$$

RESULTS

In Fig. 2 the function f_s has been plotted *vs.* ϕ and, for purposes of comparison, also the function f_c determining the corresponding case of a horizontal cylinder. (We note that the function f_c given by Nusselt[2] is in very close agreement with the function $(3)^{1/4}\,g(\phi)$ calculated by Hermann[3] for free convection without phase change.) Our solution has to be corrected to account for a finite value of the ratio $\mu\rho/\mu_l\rho_l$; however, this correction amounts to only a few percent in most cases. (For nitrogen at atmospheric pressure, $\mu\rho/\mu_l\rho_l = 2 \cdot 10^{-4}$.) Actually the vapor–liquid interface is not smooth, and results obtained from studies with horizontal cylinders show that an increase of roughly

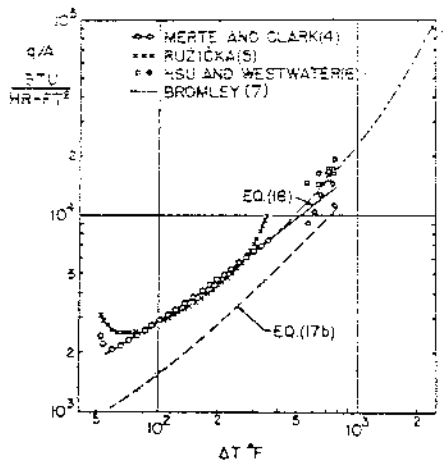

Fig. 3. Heat flow rates in film boiling of nitrogen at 1 atm and 1 g.

10% occurs. Experiments reported using liquid nitrogen[4] permit the evaluation of this influence for the sphere as well. Figure 3 shows these data (under 1 atm), and the theoretical solution, using (17b) with h_{fg} replaced by h'_{fg}. The deviation in actual heat transfer from theoretical prediction is larger than in the cylinder experiments, since the geometry of the upper half of the sphere favors separation more than that of the cylinder.

We note that film boiling heat transfer coefficients are not very different over a wide range of geometry. Cooling tests reported by Ružička[5] with vertical tubes (length about 5.7 in.) yielded nearly the same heat flux values as in the studies of Merte *et al.*[4] Hsu and Westwater[6] found that the film on vertical tubes starts to become turbulent at a height of 2.6 in. Since the heat transfer, at least in these experiments, was found to be independent of geometry it seems to be adequate to correlate these results by an equation of the form $\mathrm{Nu} = C \cdot \mathrm{Ra}^{1/3} f_1(h_{fg}/c_p \Delta T)$, or more generally,

$$\mathrm{Nu} = C \cdot \mathrm{Gr}^{1/3} f_2\left(\mathrm{Pr}, \frac{h_{fg}}{c_p \Delta T}, \frac{\mu \rho}{\mu_l \rho_l}\right) \tag{17c}$$

(f_1 and f_2 are dimensionless functions). Then the heat transfer coefficient no longer depends on geometry. The nitrogen results may be correlated by the equation

$$\mathrm{Nu} = 0.14\left(\mathrm{Ra}\frac{h'_{fg}}{c_p \Delta T}\right)^{1/3} \tag{18}$$

where the physical properties are at the mean film temperature. This equation is shown in Fig. 3 and is a modified form of that given by McAdams[8] for turbulent free convection from vertical systems.

NOTATION

A = surface area
A_c = cross-sectional area of vapor film
C = constant
c_p = specific heat at constant pressure
D = diameter of sphere
f_s = dimensionless function of ϕ for sphere
f_c = dimensionless function of ϕ for cylinder
g_0 = conversion factor
g = acceleration due to gravity
$g(\phi)$ = dimensionless function of ϕ
Gr = Grashof number
h_{fg} = latent heat of vaporization
h'_{fg} = "effective" heat of vaporization
k = thermal conductivity
Nu = Nusselt number
Pr = Prandtl number
q = heat flow rate
$q_0(\phi)$ = see (15b)
q_T = total heat removed from sphere
r = radius
R = radius of sphere
Ra = Rayleigh number, $gD^3(\rho_l - \rho)/v \alpha \rho$
T = temperature; T_s saturation temperature; T_w wall temperature
$\Delta T = T_w - T_s$
u = velocity component in x direction
v = velocity component in y direction
w_l = interfacial mass flow
x = coordinate along surface

y = coordinate normal to surface
z = dimensionless function of ϕ
α = thermal diffusivity
δ = thickness of vapor film
μ = dynamic viscosity
ν = kinematic viscosity
ρ = vapor density
ρ_l = saturated liquid density
τ = shear stress
ϕ = angular coordinate

Subscripts
l = liquid
i = interface

REFERENCES

1. W. Stenzel and G. R. Schultze, *Chemie-Ingenieur-Technik*, Vol. 30, 643 (1958).
2. W. Nusselt, *Z. VDI*, Vol. 60, 541, 569 (1916).
3. R. Hermann, "Heat Transfer by Free Convection from Horizontal Cylinders in Diatomic Gases," NACA TM 1366 (1954).
4. H. Merte and J. A. Clark, *Advances in Cryogenic Engineering, Vol. 7*, K. D. Timmerhaus (ed.), Plenum Press, New York (1962), p. 546.
5. J. Ružička, *Problems of Low-Temperature Physics and Thermodynamics*, Pergamon Press, New York (1959), p. 323.
6. Y. Y. Hsu and J. W. Westwater, *J. A.I.Ch.E.*, Vol. 4, 58 (1958).
7. L. A. Bromley, *Chem. Engr. Prog.*, Vol. 46, 221 (1950).
8. W. H. McAdams, *Heat Transmission*, 3rd ed., McGraw-Hill Book Co., New York (1954), p. 172.