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Laminar Two-Phase Boundary Layers in Natural Convection Film Boiling

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The steady laminar two-phase boundary layer considered here is generated in the gravity field by the diffusion of heat from solids, (vertical plate and a long horizontal cylinder), which are being kept at a temperature above the Leidenfrost point (heat flux minimum). Consequently film boiling takes place. Extending NUSSELT's basic work [1][2]) recently more complete similar solutions of two-phase boundary layer flow [2, 3] were presented for film condensation. In the present work film boiling solutions are given including the influence of momentum transport at the vapor-liquid interface and covering the whole range of vapor superheat up to the highest values which are physically possible in pure convective heat transfer. Helium data obtained in a continuation of previous work [4] are compared with the solutions.

1. Boundary Layer Equations for the Vertical Plate

A system of Cartesian coordinates is taken with origin at the leading edge, x axis vertically upwards, y axis normal to surface. Since the actual flow is complicated an idealized model has to be introduced. Due to this reason further the integral method (VAN KÁRMÁN-POHLHAUSEN [5]) will be applied leading quickly to results which are satisfactory within the limit given by the model. We assume incompressible fluid, constant physical properties, a smooth interface, negligible dissipation and radiation, and deal further with the special case of saturated liquid and isothermal walls. Due to the property assumption the analysis is not valid at pressures in the neighborhood of the critical point. Very low pressures, on the other hand are excluded by neglecting mean free path effects. The equations of momentum and continuity for steady laminar flow of the vapor film are

$$\varrho\, u\, \frac{\partial u}{\partial x} + \varrho\, v\, \frac{\partial u}{\partial y} = \mu\, \frac{\partial^2 u}{\partial y^2} + g\,(\varrho_L - \varrho) \tag{1}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

respectively, where u, v are the components of velocity with respect to the x, y axes; ϱ and μ are the density and coefficient of viscosity respectively; and g is the acceleration due to gravity; vapor properties are written without subscript, whereas the subscript L refers to liquid quantities. A second set of equations of momentum

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[2]) Numbers in brackets refer to References, page 218.

and continuity describes the liquid boundary layer adjacent to the vapor film. These equations may be obtained from Equations (1) and (2) by replacing the vapor properties by those of the liquid (e.g. $\varrho \to \varrho_L$).

The diffusion of heat through the vapor film is governed by the equation

$$\varrho\, c_p\, u\, \frac{\partial T}{\partial x} + \varrho\, c_p\, v\, \frac{\partial T}{\partial y} = k\, \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where T, c_p, and k are the absolute temperature, specific heat at constant pressure and heat conductivity respectively.

The boundary conditions are:

$$\left.\begin{aligned} &\text{at } y = 0 \text{ (vapor film)},\ u = v = 0;\ T = T_0\,, \\ &\text{as } y \to \infty \text{ (liquid)},\ u \to 0\,, \\ &\text{at } y = \delta,\ T = T_i \text{ (saturation temperature)}\,, \end{aligned}\right\} \tag{4}$$

where δ is the film thickness; subscript i refers to interface quantities. By inserting the hydrodynamical wall condition (4) into Equations (1) and (3) it follows that

$$\mu\left(\frac{\partial^2 u}{\partial y^2}\right)_0 = \frac{g\,(\varrho_L - \varrho)}{\varrho}; \quad k\left(\frac{\partial^2 T}{\partial y^2}\right)_0 = 0\,. \tag{5}$$

At the vapor-liquid interface certain conditions have to be fulfilled which result from the physical requirement that the tangential velocity, shear stress, and mass flow density have to be continuous. Further the interfacial heat flux through the vapor exceeds that through the liquid by an amount which is necessary to produce the interfacial mass flux of evaporated liquid. The four conditions are ($y = \delta$):

Tangential velocity

$$u_i = u_{i,L}\,. \tag{6}$$

Shear stress

$$\tau_i = \tau_{i,L}; \quad \text{where } \tau_i = \mu\left(\frac{\partial u}{\partial y}\right)_i. \tag{7}$$

Mass flow density

$$m_i = m_{i,L}; \quad \text{where } m_i = \varrho\, \frac{d}{dx} \int_0^\delta u\, dy\,. \tag{8}$$

Heat flux

$$-\,k\left(\frac{\partial T}{\partial y}\right)_i = \lambda\, m_i\,, \tag{9}$$

where λ is the latent heat of vaporization.

After having integrated the boundary layer differential equations over the film and liquid boundary layer thickness respectively the following set of equations is obtained.

Momentum equations:

Vapor film

$$\varrho\, \frac{d}{dx} \int_0^\delta u^2\, dy - m_i\, u_i = \mu\,\left(\frac{\partial u}{\partial y}\right)_i - \left(\frac{\partial u}{\partial y}\right)_0 + g\,(\varrho_L - \varrho)\,\delta\,, \tag{10}$$

liquid boundary layer

$$\varrho_L \frac{d}{dx} \int_{\delta}^{\delta_T} u^2 \, dy + m_i \, u_i = -\mu_L \left(\frac{\partial u}{\partial y} \right)_{i,L}, \tag{11}$$

where δ_T is the complete thickness of the two-phase boundary layer. We note that $\delta = \delta_T = 0$ at $x = 0$.

Energy equation:

$$\frac{d}{dx} \int_{0}^{\delta} u \, (T - T_i) \, dy = \frac{k}{\varrho \, c_p} \left[\left(\frac{\partial T}{\partial y} \right)_i - \left(\frac{\partial T}{\partial y} \right)_0 \right]. \tag{12}$$

2. Solutions of Equations (10), (11), and (12)

To take advantage of the integral method we introduce functions for velocity and temperature approximating the exact profiles [5], avoiding however the computational effort required for exact solutions to Equations (1), (2), and (3). The following functions are chosen:

Vapor velocity

$$f(\eta) = \frac{u}{\bar{u}} = \left(4 - \frac{u_i}{\bar{u}}\right) \eta + \left(2 \, \frac{u_i}{\bar{u}} - 4\right) \eta^3, \tag{13}$$

where $\eta = y/\delta$, and the reference velocity is $\bar{u} = 1/\delta \int_0^\delta u \, d\, y$.

Liquid velocity

$$F(\zeta) = \frac{u}{u_i} = 1 - 2\,\zeta + \zeta^2, \tag{14}$$

where $\zeta = (y - \delta)/(\delta_T - \delta)$.

Temperature

$$\Theta = \frac{T - T_i}{T_0 - T_i} = 1 - K_0 \, \eta + (K_0 - 1) \, \eta^3, \tag{15}$$

where K_0 is equal to the negative dimensionless temperature gradient at the wall.

Equations (13), (14), and (15) have been obtained by introducing polynomials of the third degree for the vapor profiles, and a polynomial of the second degree for the liquid velocity ratio, fulfilling the conditions (4) and (5), aside from one simplification. Instead of the hydrodynamical condition (5) a vanishing second derivative of the velocity at the wall, with respect to y, has been inserted into the function $f(\eta)$ in accordance with a satisfactory result for the corresponding one-phase problem without boiling [6]. This is expected to cause only a small error in the final heat transfer result if we consider the idealized model adopted, particularly the linearization underlying the formulation of the gravity term in Equation (1). The dimensionless parameters contained in the functions (13), (14), and (15) do account for changes of the profile shape caused by the governing physical variables and have to be evaluated subsequently.

Solutions for the unknown film thickness δ and reference velocity $\bar{u}$ are now sought which satisfy the Equations (10), (11), and (12). Since the form of the unknown functions and part of the variables involved are known [4], the desired quantities may be expressed briefly arriving finally at simple algebraic equations between the determining parameters and dimensionless variables. The vapor film functions are

$$\frac{u\, x\, \varrho}{\mu} = C_u\, (Gr_x)^{1/2} \tag{16a}$$

or

$$u = C_u \left(g\, \frac{\varrho_L - \varrho}{\varrho}\right)^{1/2} x^{1/2}, \tag{16b}$$

$$\frac{\delta}{x} = C_\delta (Gr_x)^{-1/4} \tag{17a}$$

or

$$\delta = C_\delta \left(\frac{\mu}{\varrho}\right)^{1/2} x^{1/4} \left(g\, \frac{\varrho_L - \varrho}{\varrho}\right)^{-1/4}, \tag{17b}$$

where Gr_x is the Grashof number containing the density difference or the gravity term respectively. Similarly the liquid variables are

$$u_i = C_{u,L} \left(g\, \frac{\varrho_L - \varrho}{\varrho}\right)^{1/2} x^{1/2}, \tag{18}$$

$$\delta_L = C_{\delta,L} \left(\frac{\mu_L}{\varrho_L}\right)^{1/2} x^{1/4} \left(g\, \frac{\varrho_L - \varrho}{\varrho}\right)^{-1/4}, \tag{19}$$

where $\delta_L = \delta_T - \delta$ is the liquid boundary layer thickness. Since these functions (16), (17), (18), and (19) satisfy the integral equations the parameters C_u and C_δ may be evaluated after having inserted further the profile functions (13), (14), and (15) into the Equations (10), (11), and (12). We obtain from the momentum equation (10) for the vapor film

$$C_\delta^2 \left[1 - C_u^2 \left(\frac{32}{21} - \frac{79}{84}\, \frac{u_i}{\bar{u}} + \frac{11}{84} \left(\frac{u_i}{\bar{u}}\right)^2\right)\right] = 6 \left(2 - \frac{u_i}{\bar{u}}\right) C_u \tag{20}$$

whereas the interfacial conditions (6) and (7) are rewritten as

$$C_{u,L} = C_u\, \frac{u_i}{\bar{u}}; \quad C_{\delta,L} = C_\delta \left(\frac{\mu_L\, \varrho_L}{\mu\, \varrho}\right)^{1/2} \frac{u_i}{\bar{u}}\, \frac{2}{(8 - 5\, u_i/\bar{u})}. \tag{21}$$

After having made use of (21), and the functions introduced above, the momentum equation for the liquid boundary layer (11) yields an equation containing only vapor film parameters

$$C_u^2\, C_\delta \left[\frac{\mu_L\, \varrho_L}{\mu\, \varrho}\, \frac{(u_i/\bar{u})^3}{(8 - 5\, u_i/\bar{u})} + \frac{3}{2}\, \frac{u_i}{\bar{u}}\right] = 2\, \frac{C_u}{C_\delta} \left(8 - \frac{5\, u_i}{\bar{u}}\right). \tag{22}$$

In this equation the ratio of the transport properties and densities $\mu\, \varrho/\mu_L\, \varrho_L$ appears as the physical quantity which determines the hydrodynamical conditions at the interface. It is mostly small since the liquid fluidity $1/\mu_L$ is about one order of magnitude smaller than that of the vapor, and since $\varrho/\varrho_L \sim 10^{-3}$. From the energy equation (12) the thermal parameter K_0 is found to be

$$K_0 = \frac{\left[1 + \frac{3}{140} \left(9 - \frac{u_i}{\bar{u}}\right) C_u\, C_\delta^2\, Pr\right]}{\left[1 + \frac{1}{210} \left(16 - \frac{u_i}{\bar{u}}\right) C_u\, C_\delta^2\, Pr\right]} \tag{23}$$

where Pr is the Prandtl number. The thermal interfacial condition (9) finally relates the ratio $\lambda/c_p \Delta T$, where $\Delta T = (T_0 - T_i)$, to other film parameters and the Prandtl number

$$(3 - 2 K_0) = \frac{3}{4} Pr\, C_u\, C_\delta^2 \frac{\lambda}{c_p \Delta T}. \tag{24}$$

In order to compute numerical results from our final system of Equations (20), (22), (23), and (24), use may be made of simpler functions which are valid for certain special or limiting conditions, e.g. $\mu \varrho/\mu_L \varrho_L = 0$. In the general case, for a given ratio $\mu \varrho/\mu_L \varrho_L$ and Pr, a value of C_δ may be specified which is proportional to the film thickness. After having eliminated the velocity ratio $u_i/\bar{u}$ from Equations (20) and (22) the C_u value can be found. Once a set of C_δ, C_u values has been determined it is easy to obtain K_0 which is proportional to the wall heat flux from Equation (23), and $\lambda/c_p \Delta T$ may be found from Equation (24). Thus the important quantities may be evaluated, particularly the heat transfer from the wall to the film.

3. Heat Transfer Result for the Plate

Expressing the heat transfer in terms of dimensionless quantities the local value of the Nusselt number is usually defined as

$$Nu_x = \frac{h\, x}{k} = -\left(\frac{\partial T}{\partial y}\right)_0 \frac{x}{\Delta T}, \tag{25a}$$

thus

$$Nu_x = \frac{K_0\, x}{\delta} = \frac{K_0 (Gr_x)^{1/4}}{C_\delta}, \tag{25b}$$

making use of Equation (17); h is the heat transfer coefficient.

Since $h \propto x^{-1/4}$ the mean value of Nu over a distance L is 4/3 times the local value at $x = L$, hence

$$Nu\, Gr^{-1/4} = \frac{4\, K_0}{3\, C_\delta}, \tag{26}$$

where the reference length in Gr and Nu now is L instead of x. Equation (26) is useful in the general case, however simple expressions result for certain conditions to be dealt with subsequently.

Creeping motion. If the temperature difference is small, i.e. also the heat transferred from the wall, the rate of evaporation and the mean vapor velocity likewise have low values, and inertia forces and energy convection can be neglected. Our system of equations becomes very simple, and the mean Nusselt number may be written as

$$Nu = \frac{2}{3} \left(Gr\, Pr \frac{\lambda/c_p \Delta T}{(1 - 0{\cdot}5\, u_i/\bar{u})}\right)^{1/4}. \tag{27}$$

We note that Equation (22) provides an asymptotic value of the interfacial velocity ratio

$$\left(\frac{u_i}{\bar{u}}\right)_\infty = \frac{8}{5} = 1{\cdot}6 \text{ as } \frac{\mu\, \varrho}{\mu_L\, \varrho_L} \to \infty,$$

whereas integrating Equation (1) yields a parabolic profile with

$$\left(\frac{u_i}{\bar{u}}\right)_\infty = \frac{3}{2} = 1{\cdot}5\,.$$

Zero Liquid Fluidity. In the limiting case $\mu\,\varrho/\mu_L\,\varrho_L = 0$ the interfacial velocity vanishes [Equation (22)]. We may discuss three different solutions which approximate the result from Equation (26) over a limited range of vapor superheat.

a) Creeping motion. At very small vapor superheat, according to Equation (27), is

$$Nu_0 = \frac{2}{3}\left(Gr\ Pr\ \frac{\lambda}{c_p\,\Delta T}\right)^{1/4}. \tag{28}$$

b) Moderate Superheat. If the inertia forces are neglected, energy convection, however, is taken into account the Nusselt number becomes

$$Nu = \frac{2}{3}\,(Gr\ Pr)^{1/4}\left[K_0^3\left(\frac{\lambda}{c_p\,\Delta T} + \frac{27}{35} - \frac{32}{105}\,K_0\right)\right]^{1/4}. \tag{29}$$

Approximations of this kind overestimate the heat transfer at high vapor superheat.

c) High superheat approximation. By inserting a vanishing interfacial temperature gradient into the temperature profile Equation (15), i.e. $K_0 = 3/2$, the following approximation is obtained

$$Nu = \left(\frac{\frac{22}{105}\,Gr\ Pr^{*2}}{Pr^* + \frac{80}{99}}\right)^{1/4}, \tag{30}$$

where the abbreviation has been used

$$Pr^* = Pr\left(1 + \frac{35}{11}\,\frac{\lambda}{c_p\,\Delta T}\right).$$

As $\lambda/c_p\,\Delta T \to 0$ the limiting value of Nu will be $Nu = \text{const}\ (Gr\ Pr)^{1/4}$, and in particular for $Pr = 0{\cdot}7$

$$Nu = 0{\cdot}511\ (Gr)^{1/4}\,. \tag{31}$$

4. Discussion of the Plate Solutions

The interfacial velocity ratio and the heat transfer results have been plotted in Figures 1, 2, 3, and 4 versus $\lambda/c_p\,\Delta T$ or $c_p\,\Delta T/\lambda$ respectively inserting $Pr = 0{\cdot}7$ as a value which represents a large group of substances. At creeping motion the mean vapor velocity and the momentum transfer at the interface are small. If $c_p\,\Delta T \ll \lambda$ the ratio $u_i/\bar{u}$ rises fast, with increasing $\mu\,\varrho/\mu_L\,\varrho_L$, towards its asymptotic value $(u_i/\bar{u})_\infty$. At this small vapor superheat the liquid, according to our model, behaves as a mobile fluid, and $u_i/\bar{u}$ is high over a large range of $\mu\,\varrho/\mu_L\,\varrho_L$ excluding zero liquid fluidity. At high values of $c_p\,\Delta T/\lambda$, on the other hand, the mean velocity of the vapor is high and consequently also the interfacial shear stress. In this case the liquid behaves as an immobile medium rather than a low-viscosity fluid, and the velocity ratio is small in the range $0 < \mu\,\varrho/\mu_L\,\varrho_L < 1$ which only has physical significance.

The different role the ratio $\mu\,\varrho/\mu_L\,\varrho_L$ plays at different values of $\lambda/c_p\,\Delta T$ is reflected in the heat transfer results. We note that $\mu\,\varrho/\mu_L\,\varrho_L$ has a secondary influence on the

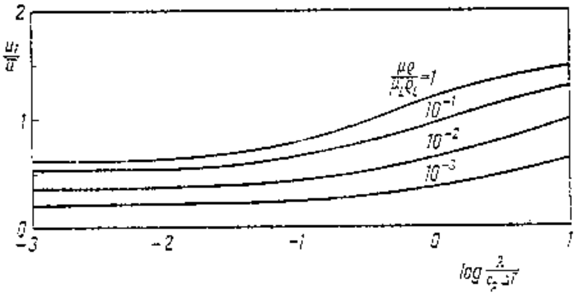

Figure 1
Interfacial velocity ratio ($Pr = 0{\cdot}7$).

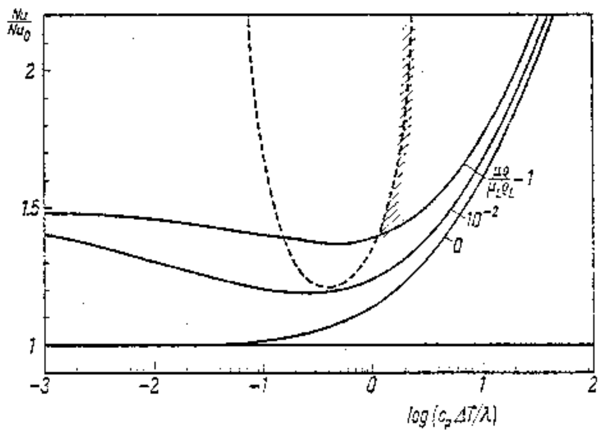

Figure 2
Heat transfer in film boiling on a vertical plate [($Pr = 0{\cdot}7$) (Nu_0 according to Equation (28)].
– – – – actual boiling curve (schematically) //////// experimental nitrogen results, Reference [10].

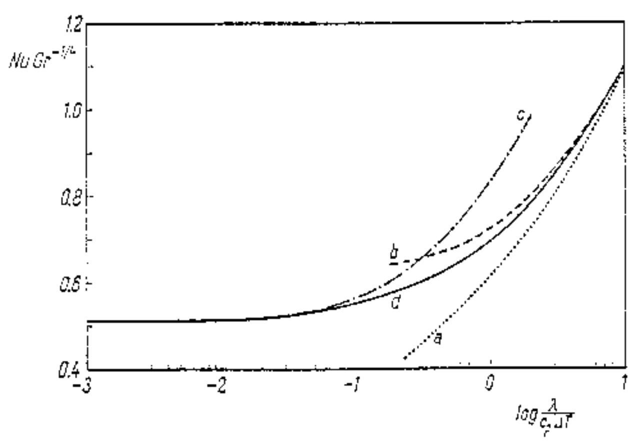

Figure 3
Heat transfer in film boiling on a vertical plate ($Pr = 0{\cdot}7$; $\mu\varrho/\mu_L\varrho_L = 0$)
a) Equation (28), *b*) Equation (29), *c*) Equation (30), *d*) Equation (26).

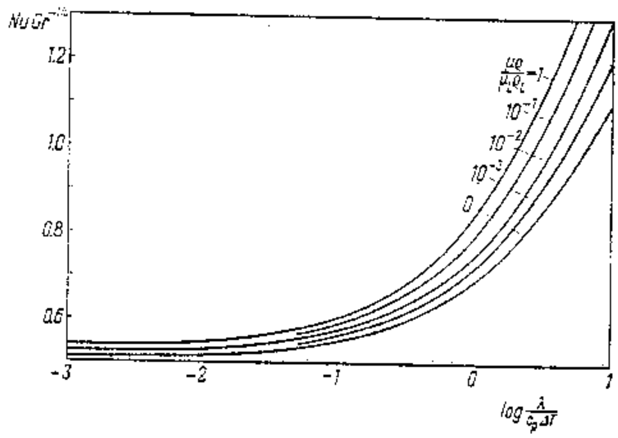

Figure 4
Plate solutions ($Pr = 0{\cdot}7$).

heat transfer particularly since values near $\mu\,\varrho/\mu_L\,\varrho_L = 1$ (critical point), have to be excluded due the property assumption underlying the analysis. In Figure 2 the Nusselt number ratio has been plotted versus $c_p\,\Delta T/\lambda$, the reference number Nu_0 being that of Equation (28). Results for small superheat, below the Leidenfrost point do not have any physical significance since transition boiling takes place. Therefore Nu has been plotted in Figures 3 and 4 versus $\lambda/c_p\,\Delta T$ to have a better presentation of moderate and high superheat results where inertia forces partially compensate the Nusselt number increase due to energy convection. In Figure 3 the different approximations arising from Equations (28), (29), and (30) are compared with the result Equation (26) inserting zero liquid fluidity. In this special case the liquid behaves as a quasi-rigid body from which molecules evaporate into the vapor film. As to be expected Equation (28) underestimates the heat transfer at large temperature differences, Equation (29) gives Nusselt numbers which are too high at $\lambda/c_p\,\Delta T < 1$, and Equation (30) overestimates Nu at $\lambda/c_p\,\Delta T > 1$. We remark that the zero liquid fluidity solution of the present integral treatment is nearly identical with a first order approximation obtained by inserting a mean excess enthalpy of the vapor of

$$\lambda_{eff} = \lambda + \frac{1}{2}\,c_p\,\Delta T \tag{32}$$

into the Equation (28) valid for creeping motion.

Figure 4 shows the Nusselt number as a function of $\lambda/c_p\,\Delta T$ for different values of the ratio $\varrho\,\mu/\mu_L\,\varrho_L$. At high superheat Nu is nearly uninfluenced by $\mu\,\varrho/\mu_L\,\varrho_L$.

5. Horizontal Cylinder

According to the boundary layer assumption of a thin film only the gravity term has to be changed in Equation (1) or (10) respectively replacing it by $g\,(\varrho_L - \varrho)/\varrho\,\sin x/R$, where R is the cylinder radius. Neglecting the inertia forces the same differential equation results which has been solved by Nusselt [1] for analogeous boundary conditions in film condensation. According to Reference [1] the over-all heat transfer coefficient of the cylinder is equal to that of a plate with height $L = 2{\cdot}85\, D$, where D is the diameter of the cylinder. At high superheat the evaluation also is easy if the inertia forces are overestimated [7] by simplifying the momentum equation and taking a small error into account. We obtain the following approximate solutions for the special case of zero liquid fluidity:

a) Creeping motion

$$Nu_D = \frac{\bar{h}\, D}{k} = 0{\cdot}513 \left(\frac{Gr_D\, Pr}{c_p\, \Delta T}\right)^{1/4}, \tag{33}$$

where the reference length in Gr is the diameter.

b) Moderate superheat

$$Nu_D\, (Gr_D\, Pr)^{-1/4} = 0{\cdot}802 \left[\frac{K_0^3}{6} \left(\frac{\lambda}{c_p\, \Delta T} + \frac{27}{35} - \frac{32}{105}\, K_0\right)\right]^{1/4}. \tag{34}$$

c) High superheat

$$Nu_D = 0{\cdot}802 \left[\frac{99}{560}\, \frac{Gr_D\, Pr^{*2}}{(Pr^* + 32/33)}\right]^{1/4}. \tag{35}$$

In particular, for $Pr = 0{\cdot}7$ and $\lambda/c_p\, \Delta T \to \infty$

$$Nu_D = 0{\cdot}383\, (Gr_D)^{1/4}. \tag{36}$$

These functions have been plotted in Figure 6 including the first order estimate which results from Equation (33) with λ replaced by λ_{eff}, Equation (32). To account for a finite liquid fluidity Figure 2 may supply small corrections.

Though some of the physical features associated with film boiling may be understood readily from the model adopted experimental support is needed due to the simplifications introduced into the analysis. The property assumption is justifiable over a large pressure range excluding near-critical and very small pressures if the vapor properties are evaluated at the arithmetic mean film temperature [8]; ($\mu\, \varrho/\mu_L\, \varrho_L$ should be evaluated at saturation conditions). The presumed smooth vapor-liquid interface, however, does not exist. Considering only inertia effects caused by interfacial wave motion the Nusselt number should decrease, however the decrease may be completely compensated by a reduction of the thermal resistance of the film. From film condensation results [9] we might expect that the increase of Nu in film boiling amounts also to a few ten percent if our approximate result is close to the rigorous solution which exactly describes film boiling with a smooth interface.

6. Comparison with Experimental Results

Laminar film boiling occurs rather infrequently in physical reality compared to turbulent motion. It prevails near leading edges, stagnation points, and on small objects. In Figure 2 for the plate an approximate curve has been plotted including transition boiling and low near-laminar heat transfer results obtained from vertical short tubes [10]. If the liquid under consideration has a high boiling point the laminar heat transfer may be completely masked by radiation.

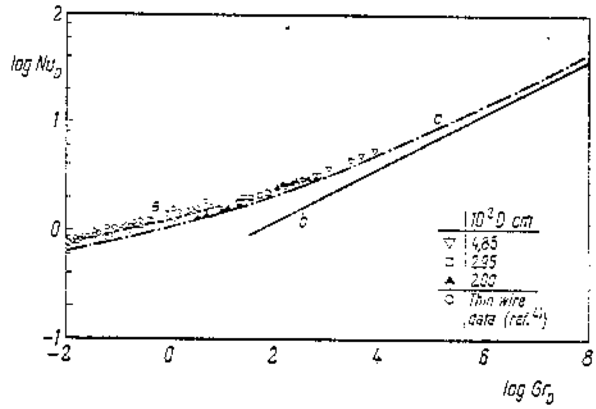

Figure 5

Film boiling on a horizontal cylinder at high vapor superheat:
a thin wire correlation Reference [4], *b* Equation (36), *c* Senftleben function, Reference [12].
Helium experiments (1 atm) at $10^2 D$ cm: ▽ 4·85, □ 2·95, ▼ 2·00, ○ thin wire data, Reference [4].

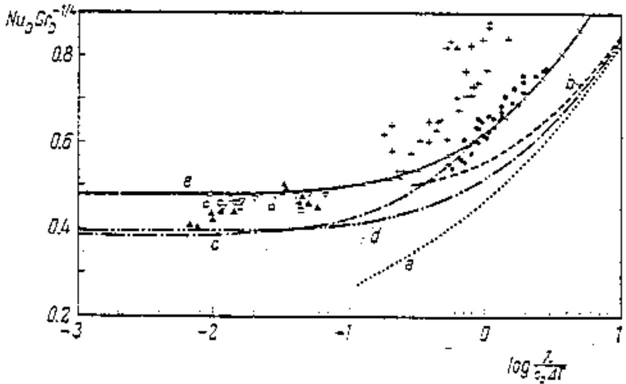

Figure 6

Film boiling on a horizontal cylinder. Solutions ($Pr = 0.7$):
a Equation (33), *b* Equation (34), *c* Equation (35), *d* Approximation Equation (33) and (32),
e BROMLEY's Equation (37).
● O_2 data, Reference [15], + H_2 data, Reference [14], He data as noted in Figure 5.

On the horizontal cylinder the flow path is short and laminar motion possible. Due to its simple form Equation (33) for creeping motion, combined with Equation (32), is very advantageous for correlating experimental results by means of a constant to be adjusted by experiments. The first boiling equation given by BROMLEY [11] is a useful correlation of this kind

$$Nu_D = 0{\cdot}62 \left(Gr_D \, Pr \, \frac{\lambda_{eff}}{c_p \, \Delta T}\right)^{1/4}. \qquad (37)$$

Thus the heat transfer is approximately 20% higher than that of the corresponding zero liquid fluidity solution. After having subtracted roughly 5% to account for the finite fluidity of the liquids used in BROMLEY's experiments an increase of Nu_D of roughly 15% results which is due to secondary influences not accounted for, e.g. interfacial waves, and errors of the integral treatment.

High superheat convective film boiling can be studied in helium experiments without remarkable radiation since the boiling point of helium is low and the heat of vaporization small. Therefore previous investigations [4] were continued in an attempt to obtain data of the boundary layer type heat transfer. To avoid prohibitive helium consumption most of the data had to be taken during slow transients after having found agreement between preliminary steady state and transient tests. (Details of the experiment have been described in Reference [4]; a minor, however economic modification was to heat the cylinders outside of the bath prior to the cool-down test.) Helium has a value of $\mu_G \varrho_G / \mu_L \varrho_L = 3 \cdot 10^{-2}$ at atmospheric pressure, and about $1{,}8 \cdot 10^{-3}$ at the λ-point.

As Figure 5 shows the boundary layer regime could not be reached completely. The Nusselt number at high superheat film boiling changes with Gr_D approximately in the same way as Nu_D of one-phase free convection with the one-phase Grashof number. For the latter case several semi-empirical correlations are known, e.g. References [6, 12, 13]. To have an estimate of the boundary layer type Nusselt number, $(Nu_D)_{red}$, a correction was applied to reduce the experimental value, $(Nu_D)_{exp}$, by means of the following equation [13]

$$(Nu_D)_{red} = \frac{(Nu_D)_{exp}}{(1 + 1{\cdot}4\, Gr_D^{-0{\cdot}21})}\,. \tag{38}$$

As Figure 6 shows the present data suggest an approximation, for $\lambda/c_p \Delta T \ll 1$,

$$Nu_D\, Gr_D^{-1/4} = \approx 0{\cdot}45 \tag{39}$$

in contrast to the previously estimated value 0·40. As Figure 3 indicates the previous approach leads to a value which is too small. The use of Equation (39) without experimental support is not recommended. This extrapolation may be in error if the laminar flow would become unstable and turbulent at small film Reynolds numbers; (compare the hydrogen results [14] in Figure 6 which are remarkably higher than the oxygen data [15]). Including the new experimental data the helium heat transfer from thin cylinders at high superheat film boiling may be represented by

$$Nu_D = 1{\cdot}2\, Gr_D^{0{\cdot}11} + 0{\cdot}09\, Gr_D^{1/3}; \quad \frac{\lambda}{c_p\, \Delta T} < 1; \quad 10^{-4} < Gr_D < 10^4\,. \tag{40}$$

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REFERENCES

[1] W. Nusselt, Z. Ver. Dtsch. Ing. *60* (1916), 541 and 569.
[2] J. C. Y. Koh, E. M. Sparrow, and J. P. Hartnett, Int. J. Heat Mass Transfer *2* (1961), 69.
[3] J. C. Y. Koh, J. Heat Transfer, Trans. ASME Ser. C, *83* (1961), 359.
[4] T. Frederking, Forschg. Geb. Ing. wes. *27* (1961), 17 and 58.
[5] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, 1955, p. 201.
[6] E. R. G. Eckert, *Introduction to the Transfer of Heat and Mass*, 2nd Ed., McGraw-Hill, 1959.
[7] G. KruJilin, J. Techn. Phys. USSR *5* (1938), 59.
[8] P. W. McFadden and R. J. Grosh, Int. J. Heat Mass Transfer *1* (1961), 325.
[9] H. Brauer, Forschg. Geb. Ing. wes. *24* (1958), 105.
[10] Y. Y. Hsu and J. W. Westwater, A. I. Ch. E. Journ. *4* (1958), 58.
[11] L. A. Bromley, Chem. Eng. Progr. *46* (1950), 211.
[12] H. Senftleben, Z. angew. Phys. *3* (1951), 361.
[13] H. Hausen, Allg. Wärmetechnik *9* (1959), 75.
[14] L. Weil, IVe Congr. Int. Chauf. Industr., Paris 1952, Nr. 210.
[15] J. T. Banchero, G. E. Barker, and R. H. Boll, Chem. Eng. Progr. Sympos. Ser., No. 17, *51* (1955), 21.

Zusammenfassung

Es wird die zweiphasige Grenzschichtströmung behandelt, die beim laminaren Filmsieden an einer vertikalen Platte und an einem langen horizontalen Zylinder unter den Bedingungen natürlicher Konvektion entsteht. In den Rechnungen wird der gesamte Bereich derjenigen Dampfüberhitzung miterfasst, in welchem rein konvektiver Wärmetransport möglich ist unter Einschluss des Impulstransports an der Phasengrenzfläche. Neue Heliummessungen und bekannte Versuchsergebnisse stimmen in befriedigender Weise mit den Rechnungen überein, wenn man berücksichtigt, dass ein vereinfachtes Strömungsmodell mit einer glatten Phasengrenzfläche vorausgesetzt ist.

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